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Water Treatment Project Observations on Use of GAC in Practice

Texas A and M Univ.
College Station

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Water Treatment Project

Observations on Use of GAC in Practice

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May 1980

WATER TREATMENT PROJECT: OBSERVATIONS ON USE OF GAC IN PRACTICE

by

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The objectives of this project were: (1) to determine if granular activated carbon (GAC) adsorption beds applied in water treatment practice slough-off organic materials during the spring warm-up and (2) to evaluate the feasibility of the dilute or low-level COD procedure for the control of GAC beds in water treatment applications.

Nine water treatment plants were studied for a period of five months during the spring of 1979. An evaluation of the COD and TOC removals versus water temperature showed that no temperature related trend in removal existed. It was found that the COD values determined by the low-level or dilute procedure did correlate well with the TOC values.

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FOREWORD

The U.S. Environmental Protection Agency was created because of increasing public and governmental concern about the dangers of pollution to the health and welfare to the American people. Noxious air, foul water, and spoiled land are tragic testimony to the deterioration of our national environment. The complexity of that environment and the interplay between its components require a concentrated and integrated attack on the problem.

Research and development is that necessary first step in problem solution and it involves defining the problem, measuring its impact, and searching for solutions. The primary mission of the Health Effects Research Laboratory in Cincinnati (HERL) is to provide a sound health effects data base in support of the regulatory activities of the EPA. To this end, HERL conducts a research program to identify, characterize, and quantitate harmful effects of pollutants that may result from exposure to chemical, physical, or biological agents found in the environment. In addition to the valuable health information generated by these activities, new research techniques and methods are being developed that contribute to a better understanding of human biochemical and physiological functions, and how these functions are altered by low-level insults.

This report provides an evaluation of nine water treatment plants which use granular activated carbon in the treatment process. Relating the water quality to the patterns of operation may provide a better understanding of what to expect when granular activated carbon is used.

R.J. Garner
Director
Health Effects Research Laboratory

ABSTRACT

The objectives of this project were: (1) to determine if granular activated carbon (GAC) adsorption beds applied in water treatment practice slough-off organic materials during the spring warm-up and (2) to evaluate the feasibility of the dilute or low-level COD procedure for the control of GAC beds in water treatment applications.

Nine water treatment plants were studied for a period of five months during the spring of 1979. An evaluation of the COD and TOC removals versus water temperature showed that no temperature related trend in removal existed. It was found that the COD values determined by the low-level or dilute procedure did correlate well with the TOC values.

This report was submitted in fulfillment of Contract C 2557-NAEX by Texas A&M University under the sponsorship of the U.S. Environmental Protection Agency. This report covers a period from February 23, 1979 through August 31, 1979, and work was completed as of December 31, 1979.

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LIST OF ABBREVIATIONS AND SYMBOLS

$\hat{\beta}_0$ - slope intercept

$\hat{\beta}_1$ - slope of regression line

H_0 - null hypothesis

H_a - alternative hypothesis

LOS - level of significance

MSE - mean square error

$S^2_{\hat{\beta}_1}$ - variance of the slope

$S_{\hat{\beta}_1}$ - standard deviation of the slope

S_p^2 - pooled variance

S_p - pooled standard deviation

SS_{xx} - sum of the squared errors of prediction

TS - test statistic

ACKNOWLEDGMENTS

This report is the product of the coordinated effort of many individuals. Among those who deserve special recognition are Dr. Harold W. Wolf, Head of the Environmental Engineering Division, Civil Engineering Department, Texas A&M University and the participating personnel at the various water treatment plants. Their assistance to the project is sincerely appreciated.

SECTION 1

INTRODUCTION

In a recently completed report[1] it was observed that the activated carbon adsorption process applied in drinking water practice may have contributed to the organic content of about 18 percent of the samples examined. About 80 percent of the increases were observed in April when the water temperature averaged 12.93°C. The average temperature was sufficiently above the average water temperature for all other samplings (3.92°C) to suggest a possible biological mechanism. The same report included a study of the monitoring methods used by the water utilities in the control of their GAC beds. Not one of the utilities reported the use of the low-level or dilute COD procedure.

The objectives of this project were:

(1) To determine if granular activated carbon (GAC) adsorption beds applied in water treatment practice slough-off organic materials during the spring warm-up and

(2) To evaluate the feasibility of the low-level COD procedure for the control of GAC in water treatment.

The scope of the study consisted of obtaining several water samples before and after GAC filters from each of nine different water treatment plants during the spring of 1979 and determining the COD and TOC values of the collected samples.

SECTION 2

CONCLUSIONS

Based on the results of the study the following conclusions have been determined:

(1) The dilute or low-level COD procedure can be used for the monitoring of GAC filters used in water treatment.

(2) There was no appreciable sloughing of bacterial growths from the filters during the spring warm-up period.

SECTION 3

EXPERIMENTAL PROCEDURES

The study involved the collection of samples from water treatment plants that used GAC filtration and the determination of both COD and TOC values for each sample.

SAMPLING

A total of 15 water treatment plants were contacted and, out of the 15 plants, 12 agreed to participate. Out of the 12 plants that agreed to participate, nine actually became involved in the study. The remaining three plants that had agreed had operational problems which prevented repeated sampling.

Sample kits were mailed to the participating plants and after the samples were collected the kits were returned by "Priority Mail" service furnished by the U.S. Postal Service. Each sampling kit mailed to the plants contained: (1) two 500 ml TOC/COD-free glass sampling bottles which contained 40 mg of sodium sulfite solution for chlorine neutralization, (2) gel-type freezing packs, (3) sampling and shipping instructions and (4) a data sheet to be completed prior to shipping. The time a sample kit was en route was less than four days. To help maximize the duration of time the samples would remain cold, water treatment plant personnel were instructed to precool the samples to 4°C and pack with frozen gel cooling packs prior to shipment. Most of the samples arrived chilled although some had reached room temperature. Actually, the arrival temperature was not important since the samples were disinfected and the previous study[1] showed that these waters had very low plate counts.

COD PROCEDURE

The COD procedure used was the low-level or dilute sample procedure outlined in Standard Methods for the Examination of Water and Wastewater, 14th edition (1976). Duplicate 50-ml samples were tested for their COD values. All glassware used was rendered COD free by placing it in a muffle furnace at 550°C for one hour.

TOC PROCEDURE

The TOC procedure used Oceanography International (O.I.) equipment

and methods. The procedure consists of two parts, ampule preparation and ampule testing which are described as follows: A 5.0-ml sample is volumetrically pipetted into a precombusted ampule covered with aluminum foil. The ampule with sample is placed in a holder attached to the O.I. ampule sealing unit. The ampule sealing unit consists of a purging unit in which purified oxygen is bubbled through the sample and an oxygen-propane microburner which seals the ampules. Then 0.25 ml of 6% phosphoric acid is added to the ampule with sample just before purging. The sample is purged with purified O_2 for 4 minutes. After 3 minutes of purging, 1 ml of saturated persulfate solution is added to the ampule. The ampule is sealed by the oxygen-propane microburner. A purified oxygen atmosphere is maintained inside the ampule during the sealing process. After all the ampules have been sealed, they are placed in a holding rack. The rack fits into a metal pressure vessel. Approximately 1 liter of distilled water is added to the pressure vessel. The vessel is sealed by a metal top that bolts on. The pressure vessel is placed in an oven at 170°C for 24 hours. The pressure vessel is allowed to cool to room temperature before the ampules are removed. The ampules are stored at room temperature until analyzed. The samples are analyzed on an O.I. ampule analyzing unit. Standard TOC samples (10.0 ppm, 7.5 ppm, 5.0 ppm, and 2.5 ppm) are run prior to the GAC samples. A linear curve is established relating an integrated machine number with the respective TOC standard. Boiled distilled water is used as dilution water for the TOC standards. The dilution water is analyzed on the ampule analyzing unit and the integrated machine number is subtracted from each of the TOC standards before the linear curve is plotted. A minimum of five samples are analyzed to obtain an average value. Once an average integrated machine number is found for a GAC sample, the respective TOC value is taken from the standard TOC curve.

SECTION 4

EXPERIMENTAL RESULTS

An average of four samples were obtained from each of the nine water treatment plants that participated.

ORGANIC REMOVAL VERSUS TEMPERATURE

Table 1 summarizes the COD and TOC removal for each sample pair, one sample being taken before and the other sample after granular activated carbon (GAC) adsorption. The average COD and TOC removal for all the data was 21 percent and 18 percent, respectively. Plots of the percent COD and TOC removal versus temperature did not show any pattern, thus, it is believed that any sloughing of microbial growths during the spring warm-up was negligible.

The chemical oxygen demand (COD) test measures the amount of oxygen needed to oxidize most organic and some inorganic compounds to carbon dioxide and water. The total organic carbon measures the organic carbon in a water. Since COD is an oxygen-demanding parameter, the COD/TOC ratio can represent the pounds of oxygen required to oxidize one pound of carbon. Thus the COD/TOC ratio is an indication of the oxidation state of the carbon.[2] A calculation of the COD/TOC ratios by linear regression using all the before and after GAC values suggests a higher oxidation state after GAC since the COD/TOC ratios were 3.43 before and 2.76 after. The rationale is that less oxygen is required for oxidation of the residual carbon after GAC treatment than before. See the Appendix for the specific statistical analysis.

Looking at the <10°C before and after GAC curves, Figures 1 and 2, the slope before is steeper than after. This means that more oxygen would be required per pound of carbon (TOC) before treatment with GAC than after (3.82 vs. 2.84). At higher temperatures (>14°C), Figures 3 and 4, the difference in slope is less (2.87 vs. 2.31) but the direction is the same (a lesser slope after GAC). Hence, the GAC treatment does satisfy some of the oxidation requirements.

The oxidation state is higher (slope is less) at warmer temperatures than at the cooler temperatures for the before GAC curves (2.87 vs. 3.82). This is as expected if biological processes in the water bodies are slowed in cooler weather. The same observation is true in comparing the after GAC curves at the two temperatures - a lesser slope at the warmer temperatures.

TABLE 1. RESULTS OF WATER TREATMENT PLANT SAMPLING

Plant No.	Water Temp. at time of Sampling	TOC Before GAC mg/l	COD Before GAC mg/l	TOC After GAC mg/l	COD After GAC mg/l	Percent COD Removed
1	2.2°	4.2	8.2	3.7	7.2	12.2
	6.7°	4.2	10.8	3.8	9.0	16.7
	7.2°	4.5	10.6	4.2	8.5	19.8
	10.0°	4.0	7.4	3.6	5.4	27.0
	12.2°	4.6	10.7	4.1	8.8	17.8
	14.4°	3.3	9.8	3.1	8.8	10.2
2	8.9°	4.3	11.7	3.3	8.6	26.5
	10.0°	3.5	8.5	3.2	6.0	29.4
	15.0°	3.8	6.4	2.6	4.2	34.4
	17.8°	3.7	7.5	2.4	4.8	36.0
3	10.0°	4.5	8.9	2.6	2.4	73.0
	15.6°	4.2	7.7	3.0	4.3	44.2
	16.7°	5.7	14.4	3.3	7.2	50.0
	19.4°	5.1	12.0	3.1	5.3	55.8
4	4.4°	3.6	4.5	3.0	4.1	8.9
	4.4°	2.8	6.5	1.9	5.9	9.2
	11.1°	3.2	4.6	2.2	4.0	13.0
	14.4°	3.2	6.5	2.8	5.2	20.0
	15.6°	3.2	6.6	2.6	6.4	3.0
	22.2°	3.3	14.7	*	*	
6	2.8°	7.3	21.2	6.5	14.1	33.5
	3.9°	7.7	24.9	6.7	20.2	18.9
	8.3°	4.8	17.0	4.7	13.9	18.2
	20.0°	5.8	18.6	5.3	13.1	29.6

(continued)

TABLE 1. RESULTS OF WATER TREATMENT PLANT SAMPLING (continued)

Plant No.	Water Temp. at time of Sampling	TOC Before GAC mg/l	COD Before GAC mg/l	TOC After GAC mg/l	COD After GAC mg/l	Percent COD Removed
8	7.0°	2.5	4.7	1.9	3.1	34.0
	17.0°	2.0	5.0	2.0	5.0	0.0
	17.0°	2.8	5.9	2.4	5.4	8.5
	23.3°	2.6	6.2	2.3	5.1	17.7
10	10.0°	3.4	10.3	broken	broken	-
	13.9°	4.5	12.5	3.3	10.1	19.2
	20.0°	3.3	8.4	2.6	7.7	8.3
	20.0°	3.0	9.1	2.2	6.4	29.7
	24.4°	3.4	5.3	*	*	
11	4.6°	3.2	7.2	2.9	7.7	+6.9
	7.4°	2.8	5.7	2.5	5.6	1.8
	24.0°	2.8	5.4	2.3	5.1	5.6
12	6.0°	broken	broken	broken	broken	-
	11.0°	3.7	7.7	3.4	7.2	6.5
	12.0°	3.5	8.0	3.3	7.2	10.0
	14.0°	3.6	7.5	3.3	5.9	21.3

* Inconclusive result

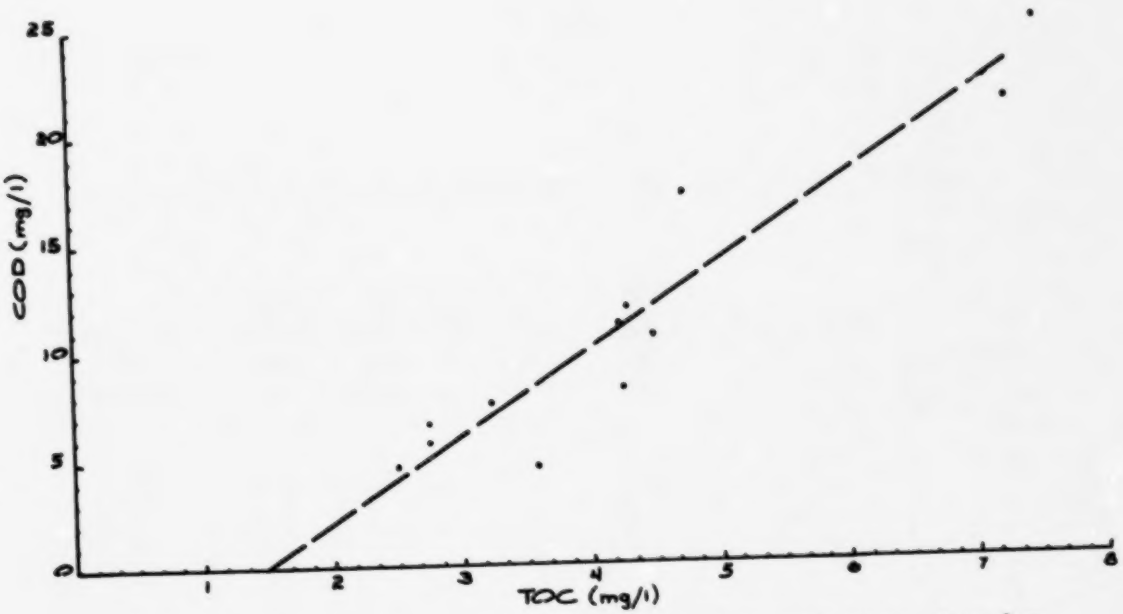

FIGURE 1: BEFORE GAC TEMP. <10°C

COD = 3.82 TOC - 5.45

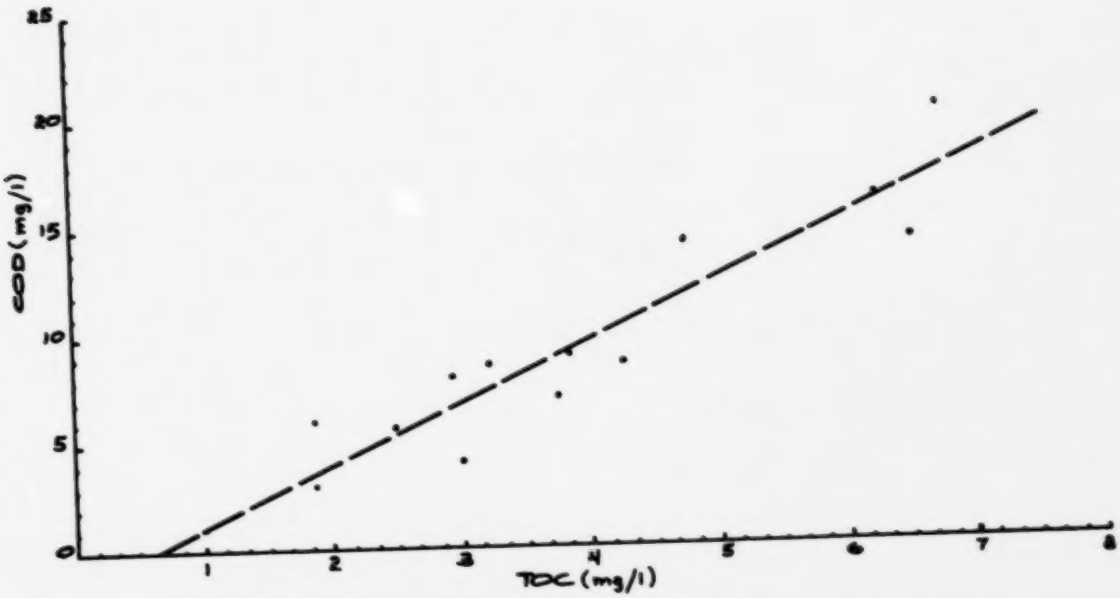

FIGURE 2: AFTER GAC TEMP. <10°C

COD = 2.84 TOC - 1.68

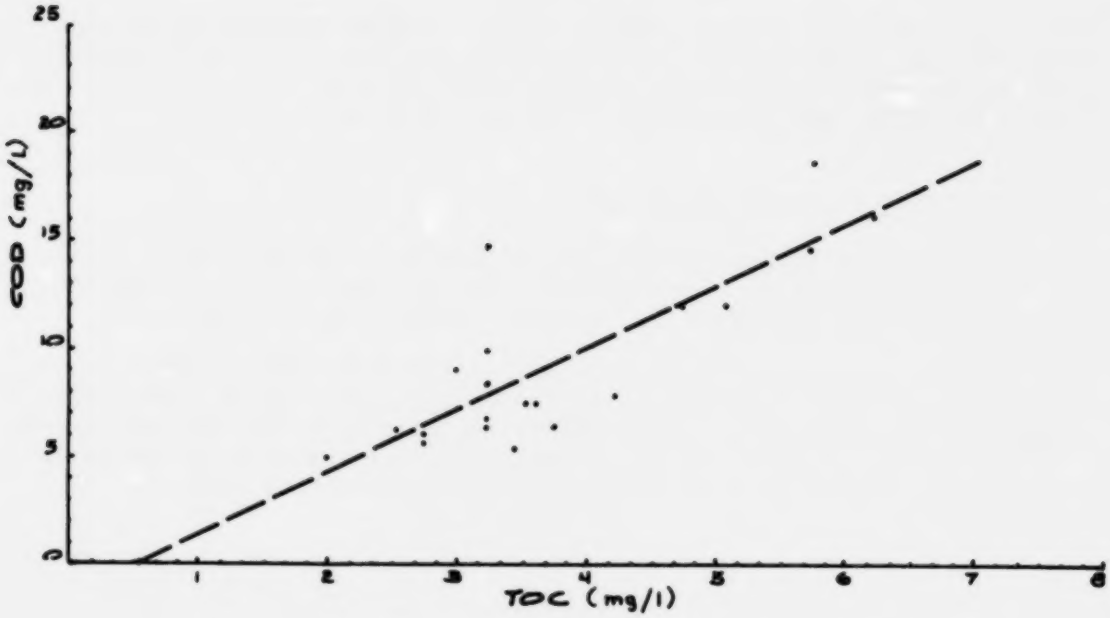

FIGURE 3: BEFORE GAC TEMP. >14°C
COD = 2.87 TOC - 1.62

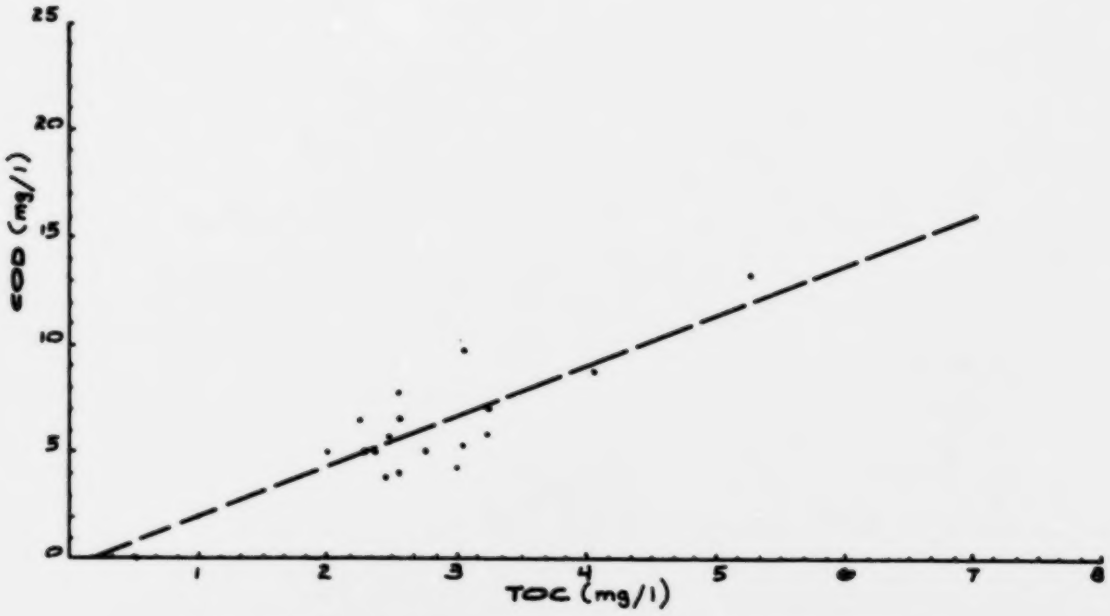

FIGURE 4: AFTER GAC TEMP. >14°C
COD = 2.31 TOC - 0.31

The amount of organic removal for all samples averaged 18 percent for TOC and 21 percent for COD. Thus, it is clear that a small amount of organic removal is accomplished by these GAC beds operating in a non-adsorptive mode since some of the beds were exhausted.

COMPARISON OF COD WITH TOC VALUES

The percent removal of COD and TOC yielded a correlation of 0.615. In the water treatment plants studied, the percent removals of COD and TOC did not show any trend as the water temperature increased.

It is recommended that the dilute or low-level COD procedure be used at each water treatment plant that employs GAC beds and which does not have TOC capability. Since TOC is recommended by EPA for the control of GAC adsorption beds, it is apparent that in the absence of costly TOC equipment, the dilute or low-level COD procedure can be used.

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2. McCarthy, J.J., "The Influence of Particle Size on Oxidation of Total, Soluble and Particulate Municipal Wastewaters", Ph.D. dissertation, Southern Methodist University, Dec. 5, 1974.

APPENDIX
STATISTICAL ANALYSES CALCULATIONS

All Data Before

n = 39

Σy	Σy^2	Σxy	Σx	Σx^2	$(x-\bar{x})^2$	$\hat{\beta}_1$	$\hat{\beta}_0$
368.60	4304.88	1618.29	151.60	643.44	54.14359	3.4256	-3.8647

$$MSE = \frac{\Sigma y^2 - \hat{\beta}_0 \Sigma y - \hat{\beta}_1 (\Sigma xy)}{n-2} = \frac{4304.88-(-3.8647)(368.60)-3.4256(1618.29)}{39-2}$$

$$MSE = 5.021$$

$$SS_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 643.44 - \frac{(151.60)^2}{39}$$

$$SS_{xx} = 54.144$$

$$S^2_{\hat{\beta}_1} = MSE/SS_{xx} = 5.021/54.144 = 0.0927$$

$$S_{\hat{\beta}_1} = \sqrt{S^2_{\hat{\beta}_1}} = \sqrt{0.0927} = 0.3040$$

Confidence Interval on $\hat{\beta}_1$ using 95% C.I., $\alpha = .05$

$$\hat{\beta}_1 \pm (t_{\alpha/2}, df_{n-2})(S_{\hat{\beta}_1})$$

$$3.4256 \pm (1.960)(0.3040)$$

$$3.4256 \pm 0.5958$$

All Data After

n = 36

Σy	Σy^2	Σxy	Σx	Σx^2	$(x-\bar{x})^2$	$\hat{\beta}_1$	$\hat{\beta}_0$
258.90	2298.43	957.73	116.10	418.87	44.4475	2.7623	-1.7168

$$MSE = \frac{\Sigma y^2 - \hat{\beta}_0 \Sigma y - \hat{\beta}_1 (\Sigma xy)}{n-2} = \frac{2298.43-(-1.7168)(258.90)-(2.7623)(957.73)}{36-2}$$

$$MSE = 2.864$$

$$SS_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 418.87 - \frac{(116.10)^2}{36}$$

$$SS_{xx} = 44.447$$

$$S^2_{\hat{\beta}_1} = MSE/SS_{xx} = 2.864/44.447 = 0.0644$$

$$S_{\hat{\beta}_1} = 0.2540$$

Confidence Interval on $\hat{\beta}_1$ using 95% C.I., $\alpha = .05$

$$\hat{\beta}_1 \pm (t_{\alpha/2}, df_{n-2})(S_{\hat{\beta}_1})$$

$$2.7623 \pm (1.960)(0.2540)$$

$$2.7623 \pm 0.4978$$

Data Before @ <10°C

n = 12

Σy	Σy^2	Σxy	Σx	Σx^2	$(x-\bar{x})^2$	$\hat{\beta}_1$	$\hat{\beta}_0$
133.00	1960.50	691.05	51.90	254.77	30.3025	3.8223	-5.4481

$$MSE = \frac{\Sigma y^2 - \hat{\beta}_0 \Sigma y - \hat{\beta}_1 (\Sigma xy)}{n-2} = \frac{1960.50-(-5.4481)(133.00)-(3.8223)(691.05)}{12-2}$$

$$MSE = 4.370$$

$$SS_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 254.77 - \frac{(51.90)^2}{12}$$

$$SS_{xx} = 30.303$$

$$S^2_{\hat{\beta}_1} = MSE/SS_{xx} = 4.370/30.303 = 0.1442$$

$$S_{\hat{\beta}_1} = 0.3798$$

Confidence Interval on $\hat{\beta}_1$ using 95% C.I., $\alpha = .05$

$$\hat{\beta}_1 \pm (t_{\alpha/2}, df_{n-2})(S_{\hat{\beta}_1})$$

$$3.8223 \pm (2.228)(0.3798)$$

$$3.8223 \pm 0.8462$$

Data After @ <10°C

n = 12

Σy	Σy^2	Σxy	Σx	Σx^2	$(x-\bar{x})^2$	$\hat{\beta}_1$	$\hat{\beta}_0$
107.90	1230.99	482.97	45.10	196.77	27.2692	2.8401	-1.6822

$$MSE = \frac{\Sigma y^2 - \hat{\beta}_0 \Sigma y - \hat{\beta}_1 (\Sigma xy)}{n-2} = \frac{1230.99-(-1.6822)(107.90)-(2.8401)(482.97)}{12-2}$$

$$MSE = 4.082$$

$$SS_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 196.77 - \frac{(45.10)^2}{12}$$

$$SS_{xx} = 27.269$$

$$S^2_{\hat{\beta}_1} = MSE/SS_{xx} = 4.082/27.269 = 0.1497$$

$$S_{\hat{\beta}_1} = 0.3869$$

Confidence Interval on $\hat{\beta}_1$ using 95% C.I., $\alpha = .05$

$$\hat{\beta}_1 \pm (t_{\alpha/2,\, df_{n-2}})\,(S_{\hat{\beta}_1})$$

$$2.8401 \pm (2.228)(0.3869)$$

$$2.8401 \pm 0.8620$$

Data Before @ >14°C

n = 17

Σy	Σy^2	Σxy	Σx	Σx^2	$(x-\bar{x})^2$	$\hat{\beta}_1$	$\hat{\beta}_0$
149.50	1560.63	589.14	61.20	238.06	17.7400	2.8715	-1.5432

$$MSE = \frac{\Sigma y^2 - \hat{\beta}_0 \Sigma y - \hat{\beta}_1 (\Sigma xy)}{n-2} = \frac{1560.63-(-1.5432)(149.50)-(2.8715)(589.14)}{17-2}$$

$$MSE = 6.642$$

$$SS_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 238.06 - \frac{(61.20)^2}{17}$$

$$SS_{xx} = 17.740$$

$$S^2_{\hat{\beta}_1} = MSE/SS_{xx} = 6.642/17.740 = 0.3744$$

$$S_{\hat{\beta}_1} = 0.6119$$

Confidence Interval on $\hat{\beta}_1$ using 95% C.I., $\alpha = .05$

$$\hat{\beta}_1 \pm (t_{\alpha/2}, df_{n-2})(S_{\hat{\beta}_1})$$

$$2.8715 \pm (2.131)(0.6119)$$

$$2.8715 \pm 1.3040$$

Data After @ >14°C

n = 15

Σy	Σy^2	Σxy	Σx	Σx^2	$(x-\bar{x})^2$	$\hat{\beta}_1$	$\hat{\beta}_0$
94.00	662.58	283.96	42.00	126.26	8.6600	2.3972	-0.4456

$$MSE = \frac{\Sigma y^2 - \hat{\beta}_0 \Sigma y - \hat{\beta}_1 (\Sigma xy)}{n-2} = \frac{662.58-(-.4456)(44.00)-(2.3972)(283.96)}{15-2}$$

$$MSE = 1.827$$

$$SS_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n} = 126.26 - \frac{(42.00)^2}{15}$$

$$SS_{xx} = 8.660$$

$$S^2_{\hat{\beta}_1} = MSE/SS_{xx} = 1.827/8.660 = 0.2110$$

$$S_{\hat{\beta}_1} = 0.4593$$

Confidence Interval on $\hat{\beta}_1$ using 95% C.I., $\alpha = .05$

$$\hat{\beta}_1 \pm (t_{\alpha/2,\, df_{n-2}})(S_{\hat{\beta}_1})$$

$$2.3972 \pm (2.160)(0.4593)$$

$$2.3972 \pm 0.9921$$

Equality of the Slopes
Between All Data Before and All Data After

Ho: $\hat{\beta}_1 - \hat{\beta}_2 = 0$ Using $\hat{\beta}_1$ = before and $\hat{\beta}_2$ = after

Ha: $\hat{\beta}_1 - \hat{\beta}_2 \neq 0$

TS:

$$t = \frac{\hat{\beta}_1 - \hat{\beta}_2}{S_\rho\{[\sum_{v}^{n_1}(X_{1v}-\bar{X}_1)^2]^{-1} + [\sum_{v}^{n_2}(X_{2v}-\bar{X}_2)^2]^{-1}\}^{1/2}} \quad df = n_1+n_2-4$$

Where $S_\rho^2 = \frac{(n_1-2)S_1^2 + (n_2-2)S_2^2}{(n_1-2) + (n_2-2)}$

$S_\rho = \sqrt{S_\rho^2}$

$S_\rho^2 = \frac{(39-2)(0.0927) + (36-2)(0.0644)}{(39-2) + (36-2)}$

$S_\rho^2 = 0.0791$

$S_\rho = 0.2813$

$$t = \frac{3.4256 - 2.7623}{0.2813[(54.1436)^{-1} + (44.4475)^{-1}]^{1/2}}$$

$t = 11.650$ $df = 39 + 36 - 4 = 71$

Concl. - the slopes are different @ LOS of $\rho<.005$

Equality of the Slopes
Between Before <10°C and After <10°C

Ho: $\hat{\beta}_1 - \hat{\beta}_2 = 0$ Using $\hat{\beta}_1$ = before <10°C and $\hat{\beta}_2$ = after <10°C

Ha: $\hat{\beta}_1 - \hat{\beta}_2 \neq 0$

$$S_\rho^2 = \frac{(12-2)(0.1442) + (12-2)(0.1497)}{(12-2) + (12-2)}$$

$S_\rho^2 = 0.1470$

S_ρ 0.3833

$$t = \frac{3.8223 - 2.8401}{0.3833[(30.3025)^{-1} + (27.2692)^{-1}]^{1/2}}$$

$t = 9.710$ $\quad df = 12 + 12 - 4 = 20$

Concl. - the slopes are different @ LOS of $\rho<0.005$

Equality of the Slopes
Between Before >14°C and After >14°C

Ho: $\hat{\beta}_1 - \hat{\beta}_2 = 0$ Using $\hat{\beta}_1$ = before >14°C and $\hat{\beta}_2$ = after >14°C

Ha: $\hat{\beta}_1 - \hat{\beta}_2 \neq 0$

$$S_\rho^2 = \frac{(17-2)(0.3744) + (15-2)(0.2110)}{(17-2) + (15-2)}$$

$$S_\rho^2 = 0.2985$$

$$S_\rho = 0.5464$$

$$t = \frac{2.8715 - 2.3972}{(0.5464)[(17.7400)^{-1} + (8.6600)^{-1}]^{1/2}}$$

$t = 2.094$ $\quad df = 17 + 15 - 4 = 28$

Concl. - the slopes are different @ LOS of $\rho<0.023$

Equality of the Slopes
Between Before @ <10°C and Before >14°C

Ho: $\hat{\beta}_1 - \hat{\beta}_2 = 0$ Using $\hat{\beta}_1$ = <10°C and $\hat{\beta}_2$ >14°C

Ha: $\hat{\beta}_1 - \hat{\beta}_2 \neq 0$

$$S_\rho^2 = \frac{(12-2)(0.1442) + (17-2)(0.3744)}{(12-2) + (17-2)}$$

$$S_\rho^2 = 0.2823$$

$$S_\rho = 0.5313$$

$$t = \frac{3.8223 - 2.8715}{0.5313[(30.3025)^{-1} + (17.7400)^{-1}]^{1/2}}$$

$t = 5.990$ $df = 12 + 17 - 4 = 25$

Concl. - the slopes are different @ LOS of $\rho<.005$

Equality of the Slopes
Between After <10°C and After > 14°C

H_o: $\hat{\beta}_1 - \hat{\beta}_2 = 0$ Using $\hat{\beta}_1$ = <10°C and $\hat{\beta}_2$ >14°C

H_a: $\hat{\beta}_1 - \hat{\beta}_2 \neq 0$

$$S_\rho^2 = \frac{(12-2)(0.1497) + (15-2)(0.2110)}{(12-2) + (15-2)}$$

$$S_\rho^2 = 0.1843$$

$$S_\rho = 0.4294$$

$$t = \frac{2.8401 - 2.3972}{(0.4294)[(27.2692)^{-1} + (8.66)^{-1}]^{1/2}}$$

$t = 2.644$ $df = 12 + 15 - 4 = 23$

Concl. - the slopes are different @ LOS of $\rho<0.0077$